

Before firing

After extinguishment

Fig. 1 Typical segmented 2-in. cylindrical perforated motor extinguished with water injection.

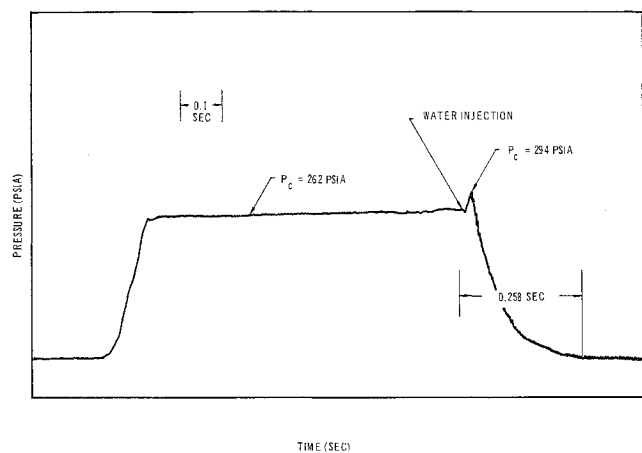


Fig. 2 Typical pressure-time trace of slotted (2-in. cylindrical perforated) motor extinguished with water injection.

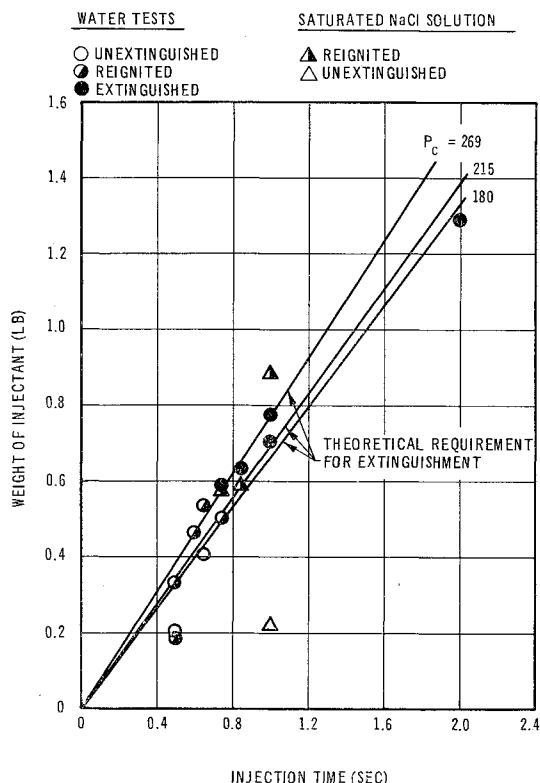


Fig. 3 Injectant weight as a function of injection time for motors operating at 180–269 psia chamber pressure.

The minimum water requirement to obtain complete extinguishment was 0.708 lb H₂O/lb of mixture; this value was in accord with the theoretical weight ratio of 0.715 lb H₂O/lb of mixture. The injection time appears to be critical in that the water must have sufficient residence time in the motor to absorb the residual heat from the propellant gases and propellant surface to prevent reignition.

References

- ¹ Eckert, E. R. G., "Engineering relations for heat transfer and friction in high-velocity laminar and turbulent boundary-layer flow over surfaces with constant pressure and temperature," *Trans. Am. Soc. Mech. Engrs.* **78**, 1273–1283 (1956).
- ² Summerfield, M., Sutherland, G. S., Webb, M. J., Taback, H. J., and Hall, K. P., "Burning mechanism of ammonium perchlorate propellants," *ARS Progress in Astronautics and Rocketry: Solid Propellant Rocket Research*, edited by M. Summerfield (Academic Press, New York, 1960), Vol. I, p. 142.

Heat Diffusion from Line Source into Mixing Region of Two Parallel Streams

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THE basic problem under consideration is illustrated by Fig. 1. Depicted here are two parallel streams of unequal but uniform velocities interacting in a turbulent mixing region downstream of a thin dividing plate. A line heat source is placed immediately downstream from the dividing plate.

The author's interest in this problem resulted from his investigation of heat transfer for separated flow past relatively deep cavities.¹ The configuration shown in Fig. 1 was one mechanism in the over-all heat-transfer model.

The velocity profile for this configuration has been rather well established² to be represented by

$$\Phi = u/u_a = \frac{1}{2}[(1 + \Phi_b) + (1 - \Phi_b) \operatorname{erf} \eta] \quad (1)$$

where

- u = $u(\eta)$ = x velocity component
- u_a = approach velocity of faster moving stream
- u_b = approach velocity of slower moving stream
- Φ_b = u_b/u_a
- η = $\sigma y/x$
- σ = $\sigma(\Phi_b)$ = empirical similarity parameter
- x, y = coordinates in intrinsic coordinate system, displaced from physical coordinate system (X, Y) by amount η_M

$$\operatorname{erf} \eta = \frac{2}{\pi^{1/2}} \int_0^\eta e^{-s^2} ds$$

Equation (1) assumes similar velocity profiles, and the same assumption is employed in the present development for the temperature profiles. η_M is determined from momentum considerations, and the reader is referred to Ref. 2 for the details of this matter. For the mixing of an incompressible fluid with a quiescent wake, the similarity parameter (σ) has been found to be 12. Korst² recently has formulated an expression applicable for any value of Φ_b , but there is still much uncertainty about the value of $\sigma(\Phi_b)$.

The differential equation governing the excess temperature (T_{ex}) in the mixing region resulting from the line heat source is

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the energy equation, simplified by boundary-layer considerations

$$c_p \rho u \frac{\partial T_{ex}}{\partial x} + c_p \rho v \frac{\partial T_{ex}}{\partial y} = \frac{\partial}{\partial y} \left(K_t \frac{\partial T_{ex}}{\partial y} \right) \quad (2)$$

Here v is the vertical velocity component and K_t is the turbulent heat-exchange coefficient. Assuming a turbulent Prandtl number of unity, incompressible flow, and K_t independent of y , results in the following expression:

$$u(\partial T_{ex}/\partial x) + v(\partial T_{ex}/\partial y) = \epsilon(\partial^2 T_{ex}/\partial y^2) \quad (2a)$$

where $\epsilon = \epsilon(x)$ is the kinematic eddy viscosity. Regarding ϵ , a form originally suggested by Goertler³ and modified by H. Korst² will be used. This is expressed as

$$\epsilon = xu_a/4\sigma^2 \quad (3)$$

The formulation of the vertical velocity component v is derived from the continuity equation

$$(\partial u/\partial x) + (\partial v/\partial y) = 0 \quad (4)$$

Inasmuch as $u = u(\eta)$, and because v is related to u through Eq. (4), it follows that $v = v(\eta)$. This enables Eq. (4) to be written as an ordinary differential equation:

$$\sigma(dv/d\eta) = \eta(du/d\eta) \quad (4a)$$

Equation (4a) can be integrated by parts to yield

$$\frac{\sigma v}{u_a} = \frac{\eta u}{u_a} - \int_{\eta_{ref}}^{\eta} \frac{u}{u_a} d\eta + \text{const} \quad (5)$$

The boundary condition on this equation is supplied from the fact that along the streamline dividing the two streams (the j streamline, which emanates as a ray from the termination of the dividing plate), the following relationship holds:

$$v_j/u_j = y_j/x_j \text{ or } \sigma(v_j/u_a) = \Phi_j \eta_j \quad (6)$$

where values of $\eta_j = \eta_j(\Phi_b)$ are known from the calculations of Korst.² Equation (6) thus allows Eq. (5) to be expressed in the following form:

$$\frac{\sigma v}{u_a} = \eta \Phi - \int_{\eta_j}^{\eta} \Phi d\eta \quad (5a)$$

It is now possible to consider a solution to Eq. (2a), inasmuch as expressions for all the terms in this equation have been presented. As previously mentioned, a similar solution form for T_{ex} is assumed. This assumption is justified by the author in Ref. 1, and results in the following form for T_{ex} :

$$T_{ex} = k\Theta(\eta)/x \quad (7)$$

where Θ is the remaining function to be determined.

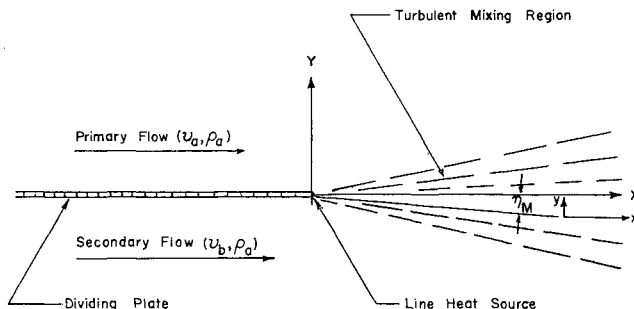


Fig. 1 Two-stream mixing and heat diffusion model. X and Y refer to a reference system of coordinates, i.e., lined up with the dividing plate. x and y refer to an intrinsic system of coordinates, which are displaced by the amount η_M from the reference system.

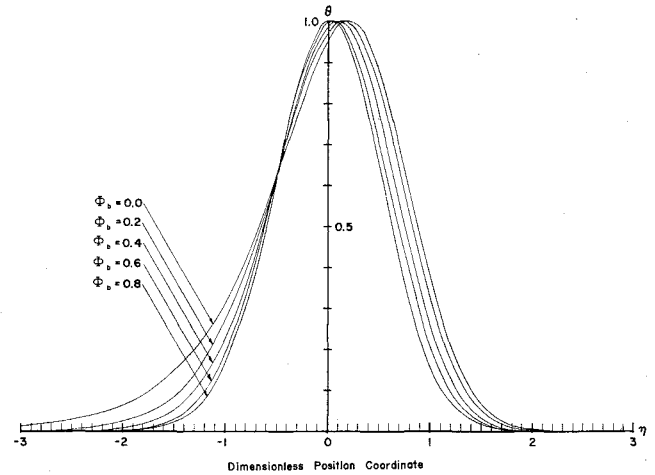


Fig. 2 Dimensionless temperature profiles θ for configuration of Fig. 1.

Substitution of Eqs. (1, 3, 5a, and 7) into Eq. (2a) results in the following differential equation for $\Theta = \Theta(\eta; \Phi_b)$:

$$\ddot{\Theta} + 4\dot{\Theta} \left\{ \int_{\eta_j}^{\eta} \frac{1}{2} [(1 + \Phi_b) + (1 - \Phi_b) \text{erf}\eta] d\eta \right\} + 4\Theta \left\{ \frac{1}{2} [(1 + \Phi_b) + (1 - \Phi_b) \text{erf}\eta] \right\} = 0 \quad (8)$$

Equation (8) is recognized as an ordinary, linear, second-order differential equation with variable coefficients. It was felt that no closed form solution to this equation would be found, and so $\Theta = \Theta(\eta; \Phi_b)$ was determined by programming an appropriate Runge-Kutta numerical integration technique onto a digital computer.

The numerical integration was started at $\eta = \eta_j$ and proceeded to $\eta = +\infty$ and to $\eta = -\infty$. The choice of starting the integration at η_j had two advantages. First, it simplified the evaluation of the coefficient of Θ in Eq. (8). Second, it was reasoned that $\dot{\Theta}|_{\eta_j} = 0$, since the line source introduces heat along the j streamline, and thus this streamline must have the highest temperature (zero slope of temperature profile). This knowledge of $\dot{\Theta}|_{\eta_j}$ provides one of the two boundary conditions needed for the solution of Eq. (8). The other condition was provided by assigning the value of $\Theta|_{\eta_j} = 1$, which is permissible, since the constant k in Eq. (7) is unspecified.

The results obtained by this integration of Eq. (8) are shown in Fig. 2 by the plots of

$$\Theta(\eta; \Phi_b) = T_{ex}(\eta; \Phi_b)/T_{ex}(\eta_j; \Phi_b) \quad (9)$$

In experimenting with the solution to Eq. (8) on the digital computer, it was discovered that the boundary condition of $\dot{\Theta}|_{\eta_j} = 0$ was the only possible condition that would give solutions for Θ which asymptotically approach zero for $\eta \rightarrow +\infty$ as well as for $\eta \rightarrow -\infty$. Physical reasoning dictates that the proper solution for Θ must do as described, and thus support is given to the solutions indicated in Fig. 2.

Simultaneously with determining Θ , a quantity proportional to the heat flux passing between the j streamline and any arbitrary streamline was also computed. This heat flux quantity (HF) is defined as

$$HF = HF(\eta; \Phi_b) = \int_{\eta_j}^{\eta} \Phi \Theta d\eta \quad (10)$$

Finally, the percentage of the total diffused heat carried into the upper stream (or lower stream) can be determined as indicated by Eq. (11). This quantity [$\pi = \pi(\Phi_b)$] is presented in Fig. 3:

$$\pi = \pi(\Phi_b) = \int_{\eta_j}^{\infty} \Phi \Theta d\eta / \int_{-\infty}^{\infty} \Phi \Theta d\eta \quad (11)$$

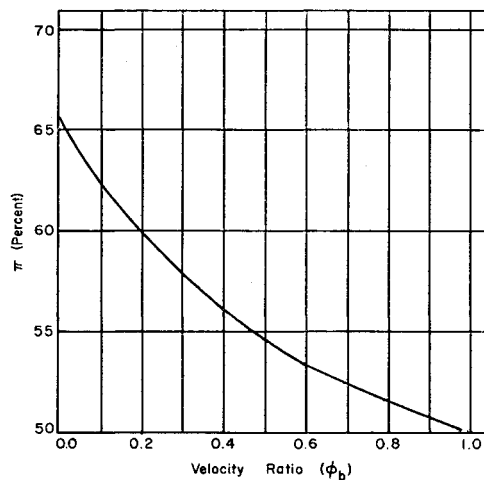


Fig. 3 Percentage of total diffused heat π contained in faster stream.

References

- ¹ Miles, J. B., "Stanton number for separated turbulent flow past relatively deep cavities," Ph. D. Thesis, Mechanical Engineering Dept., Univ. of Illinois (1963).
- ² Korst, H. H. and Chow, W. L., "The compressible turbulent jet mixing between two streams at constant pressure," Engineering Experiment Station, Univ. of Illinois, M. E. TN 393-2 (1963).
- ³ Goertler, H., "Berechnung von aufgaben der freien turbulenz auf grund eines neuen naherungsansatzes," Z. Angew. Math. Mech. 22, 244-254 (1942).

Temperature Yield Strength Correlation in Hypervelocity Impact

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Nomenclature

- V_c = crater volume
 V_p = projectile volume
 V_0 = impact velocity
 C_0 = sound speed in target

Introduction

A PREREQUISITE for the complete formulation of the effects of high-velocity particles impacting on a thick target is the understanding of the basic physical processes involved. Each of the large number of variables which appear to influence the resulting crater dimensions and their interdependence must be evaluated.

Although it is generally recognized that some mechanical strength property of the target and its temperature dependence is important in the cratering process, no experiments have tied down this temperature strength correlation. An investigation was made to determine whether there is an exact correlation between the target yield strength and the target temperature.

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Experiment

It has generally been known that the crater size is dependent upon on the mechanical strength of the target. Experiments also have been conducted demonstrating that heating the target caused larger craters to result from impacts.¹ It also has been pointed out that, when the temperature is varied, anomalies in the crater dimensions appear at temperatures where anomalies occur in the strength of the target material.² Others have shown that a favorable graphical comparison exists between the cratering efficiency and the target tensile strength as a function of temperature.³ The question arises then as to whether an exact correlation in crater size exists between the target temperature and a mechanical property, such as the yield strength. For example, consider cratering in two targets of the same material but different yield strengths. If the yield strength of the two targets were made equal by raising the temperature of the higher yield strength target, would equal size craters be obtained for a given impact?

The experiment consisted of firing $\frac{1}{4}$ -in.-diam aluminum spheres into 2-in.-thick, 8-in.-diam aluminum targets. The targets were of 7075-T6, 2017-T4, and 7075-0 aluminum, having room temperature yield strengths of 68,400, 38,700, and 17,000 psi, respectively. (These values were obtained from yield strength tests made on samples of the target material stock.) A range of impact velocities was covered for each of the three different targets at room temperature, and the resulting craters were measured. Impacts were made on four 7075-T6 targets, two of which were heated to 380°F and two to 500°F, having true yield strengths of 40,500 and 20,500 psi, respectively, and on one 2017-T4 target heated to 357°F, having a true yield strength of 25,500 psi. (These yield strength values also were obtained from curves constructed with data from yield strength tests made over a range of temperatures.)

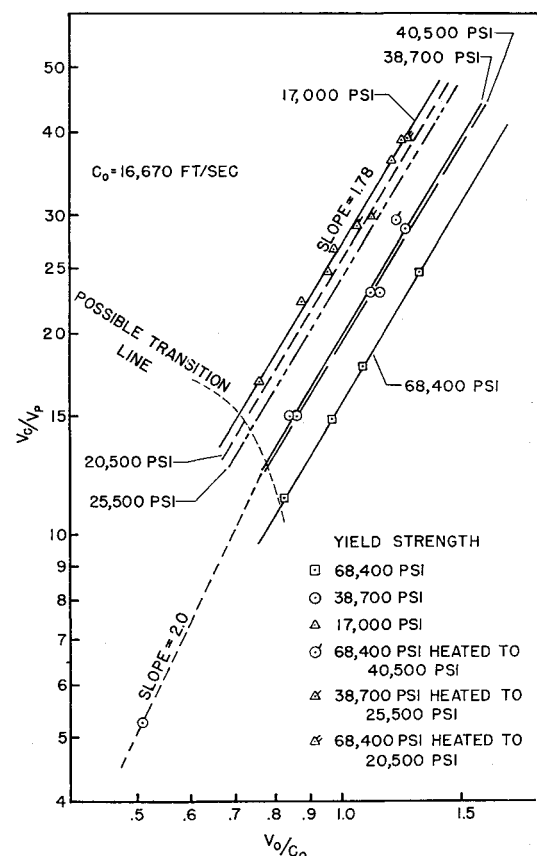


Fig. 1 Yield strength correlation.